Where ϕ is another function.

To solve it we have .

$C =$ constant at const. temp

$\therefore \quad d(C^2) = 0$ from eqn (A)

or, $d(u^2+v^2+w^2) = 0$

or, $u\,du + v\,dv + w\,dw = 0$ — (2)

Now taking differential of eqn (1) both sides, we have

$$f'(u)\,du\,f(v)\,f(w) + f'(v)\,dv\,f(u)\,f(\omega)$$
$$+ f'(\omega)\,dw\,f(u)\,f(v) = 0$$

where dashes represents Ist derivation

Dividing above eqn by $f(u)\,f(v)\,f(\omega)$ we have

$$\frac{f'(u)}{f(u)}\,du + \frac{f'(v)}{f(v)}\,dv + \frac{f'(\omega)}{f(\omega)}\,dw = 0 \quad \text{—— (3)}$$

Multiplying eqn (2) by constant β both sides we get—

$$\beta u\,du + \beta v\,dv + \beta w\,dw = 0 \quad (3\,i)$$

Now combining (3) & (3i) by Laplaces method of undetermined multipliers we have.

$$\left(\frac{f'(u)}{f(u)} + \beta u\right)du + \left(\frac{f'(v)}{f(v)} + \beta v\right)dv + \left(\frac{f'(w)}{f(w)} + \beta w\right)dw$$
$$= 0 \quad \text{—— (4)}$$

identity
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Now this expression is an identity.
Hence separate terms must vanish.
Taking 1st Term we get

$$\left(\frac{f'(u)}{f(u)} + \beta u\right) du = 0$$

or, $$\frac{f'(u)}{f(u)} du = -\beta u\, du$$

Integrating we get

$$\log f(u) = -\beta \frac{u^2}{2} + \text{Const.}$$

where
$\log_e e = 1$
$a = \text{Const.}$
$\log_e 1 = 0$
$\log m^n = n \log m$

$$= -\frac{\beta u^2}{2}\log_e e + \log a$$

$$= \log e^{-\frac{\beta u^2}{2}} + \log a$$

$$= \log e^{-bu^2} + \log a$$

where $b = \beta/2$

or $$\log f(u) = \log a e^{-bu^2}$$

~~Similarly~~ or $$f(u) = a e^{-bu^2}$$

Similarly $$f(v) = a e^{-bv^2}$$

and $$f(w) = a e^{-bw^2}$$

These function are determined and we have

$$f(u) f(v) f(w) = a^3 e^{-b(u^2+v^2+w^2)}$$

And the number of molecules lying between velocity range u and $u+du$, v and $v+dv$, w & $w+dw$ in elementary volume is given by

$$dn = n\, a^3\, e^{-b(u^2+v^2+w^2)}\, du\, dv\, dw$$

The Constant $a = \sqrt{b/\pi}$ and $b = \frac{m}{2KT}$

where K = Boltzmann constant
T = absolute temp.
m = mass of a molecule

Hence
$$dn = n \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-\frac{m}{2}\left(\frac{u^2+v^2+w^2}{KT}\right)} du\, dv\, dw \qquad (5)$$

We can now obtain the number of molecules lying between c & $c+dc$. Where we have the volume $du\, dv\, dw$ is velocity space and in place of this elementary volume. The volume element in spherical polar co-ordinates (r, θ, ϕ)

$$= dc \cdot c\, d\theta \cdot c \sin\theta\, d\phi$$

and may be substituting for $du\, dv\, dw$ in eqn (5) and velocities are now between c & $c+dc$, θ & $\theta+d\theta$, ϕ & $\phi+d\phi$.

To find the total no. of molecules between c & $c+dc$, we have to integrate the expression for all possible value of ϕ & θ.

$[\phi]_0^{2\pi}$
$\pi - 0 = 2\pi$
$d\theta = [-\cos\theta]_0^{\pi}$
$[-\cos\theta]$
$[-1-1]$
$[-2]$
$\to 2$

Hence
$$dn_c = n\, a^3\, e^{-bc^2}\, c^2\, dc \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta$$

$$dn_c = 4\pi n\, a^3\, e^{-bc^2}\, c^2\, dc \qquad (6)$$

which is the distribution law of Maxwell.

Which is the distribution law of Maxwell.
Now putting $bc^2 = x^2$
we have
$$dn_c = 4n\,\pi^{-1/2}\,x^2\,e^{-x^2}\,dx$$

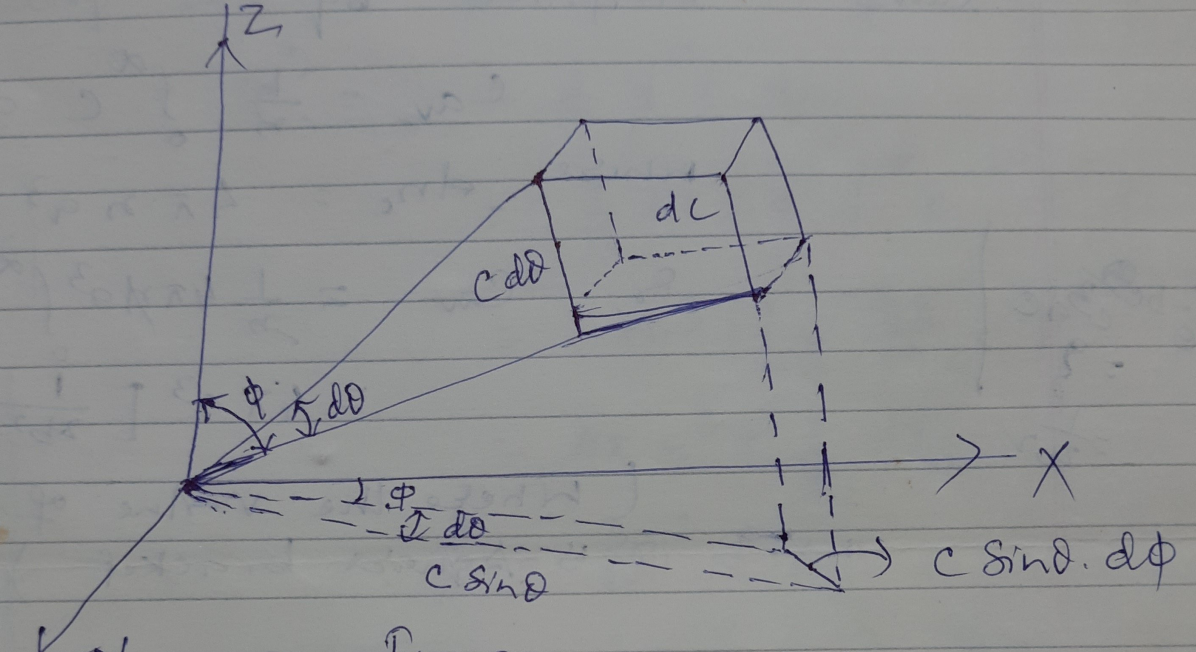
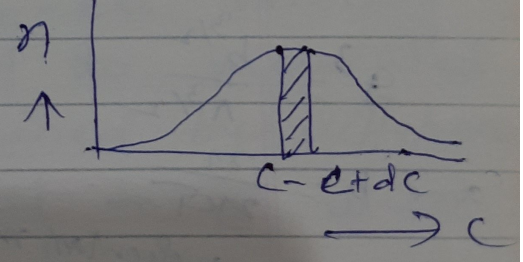


fig 2

If we plot the function $y = 4\pi^{-1/2} e^{-x^2} x^2$ ~~$y = 4\pi^{-1/2} e^{-x^2} x^2$~~ ~~graph~~ against x, we get the Maxwellian curves as shown in fig below.

And the number of molecules dn_c whose speed lies between u & $u+du$ is proportional to shadowed area.



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